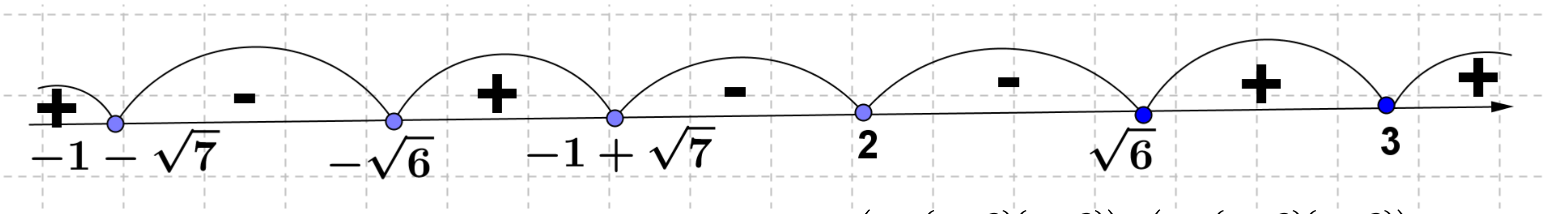
Классная работа. Множества. Решение.

Найдите множество решений неравенства:

- $\left|\frac{x^2-3x}{x-2}\right| \geq |x^2-9| \Leftrightarrow \left(\frac{x^2-3x}{x-2}\right)^2 \geq (x^2-9)^2 \Leftrightarrow \left(\frac{x^2-3x}{x-2}\right)^2 - (x^2-9)^2 \geq 0 \Leftrightarrow$



$$\Leftrightarrow (x-3)^2 \cdot \left(\frac{x}{x-2}+x+3\right) \cdot \left(\frac{x}{x-2}-x-3\right) \geq 0 \Leftrightarrow (x-3)^2 \cdot \left(\frac{x+(x+3)(x-2)}{x-2}\right) \cdot \left(\frac{x-(x+3)(x-2)}{x-2}\right) \geq 0$$

$$\Leftrightarrow$$

$$\Leftrightarrow \frac{(x-3)^2}{(x-2)^2} \cdot (x^2+2x-6) \cdot (-x^2+6) \geq 0 \Leftrightarrow \frac{(x-3)^2}{(x-2)^2} \cdot \left(x-(-1-\sqrt{7})\right) \cdot \left(x-(-1+\sqrt{7})\right) \cdot (x-\sqrt{6})(x+\sqrt{6}) \leq 0$$

$$\Leftrightarrow$$

Ответ: $[-1-\sqrt{7}; -\sqrt{6}] \cup [-1+\sqrt{7}; \sqrt{6}] \backslash \{2\} \cup \{3\}$.

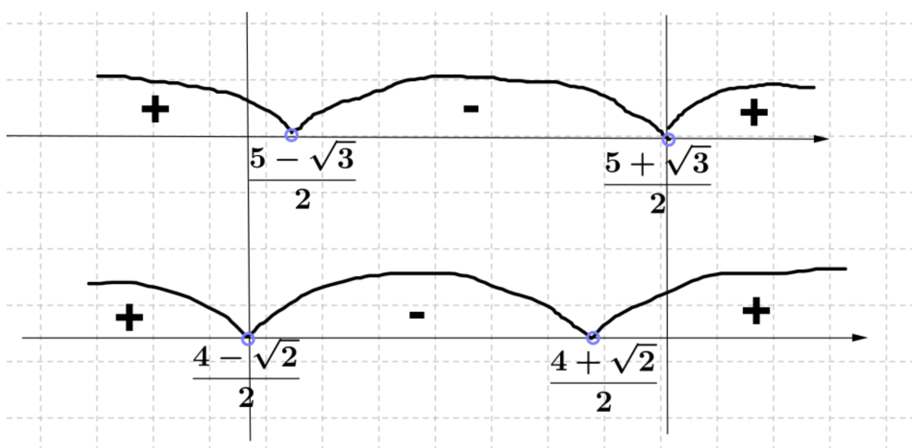
- $|x-2| \geq 2x^2-9x+9 \Leftrightarrow \left[\begin{array}{l} x-2 \geq 2x^2-9x+9 \\ x-2 \leq -2x^2+9x-9 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 2x^2-10x+11 \leq 0 \\ 2x^2-8x+7 \leq 0 \end{array}\right. \Leftrightarrow$

$$\left[\begin{array}{l} 2 \cdot \left(x-\frac{5+\sqrt{3}}{2}\right) \cdot \left(x-\frac{5-\sqrt{3}}{2}\right) \leq 0 \\ 2 \cdot \left(x-\frac{4+\sqrt{2}}{2}\right) \cdot \left(x-\frac{4-\sqrt{2}}{2}\right) \leq 0 \end{array}\right. \Leftrightarrow$$

✓ $2x^2-10x+11=0 \Leftrightarrow \left[\begin{array}{l} x=\frac{5+\sqrt{25-2\cdot 11}}{2} \\ x=\frac{5-\sqrt{25-2\cdot 11}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{5+\sqrt{25-22}}{2} \\ x=\frac{5-\sqrt{25-22}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{5+\sqrt{3}}{2} \\ x=\frac{5-\sqrt{3}}{2} \end{array}\right.$.

✓ $2x^2-8x+7=0 \Leftrightarrow \left[\begin{array}{l} x=\frac{4+\sqrt{16-2\cdot 7}}{2} \\ x=\frac{4-\sqrt{16-2\cdot 7}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{4+\sqrt{16-14}}{2} \\ x=\frac{4-\sqrt{16-14}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{4+\sqrt{2}}{2} \\ x=\frac{4-\sqrt{2}}{2} \end{array}\right.$

$$\Leftrightarrow \left[\begin{array}{l} \frac{5-\sqrt{3}}{2} < x < \frac{5+\sqrt{3}}{2} \\ \frac{4-\sqrt{2}}{2} < x < \frac{4+\sqrt{2}}{2} \end{array}\right. \Leftrightarrow \frac{4-\sqrt{2}}{2} < x < \frac{5+\sqrt{3}}{2}.$$

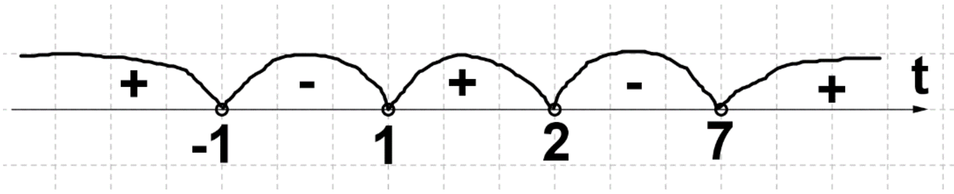


Ответ: : $\left[\frac{4-\sqrt{2}}{2}; \frac{5+\sqrt{3}}{2}\right]$.

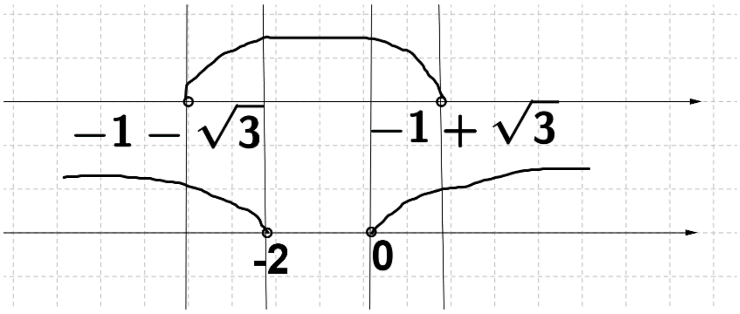
- $\frac{12}{x^2+2x}-\frac{3}{x^2+2x-2}>1 \Leftrightarrow \begin{cases} x^2+2x-1=t \\ \frac{12}{t+1}-\frac{3}{t-1}>1 \end{cases} \Leftrightarrow \begin{cases} x^2+2x-1=t \\ \frac{12(t-1)-3(t+1)-t^2+1}{(t+1)(t-1)}>0 \end{cases} \Leftrightarrow$

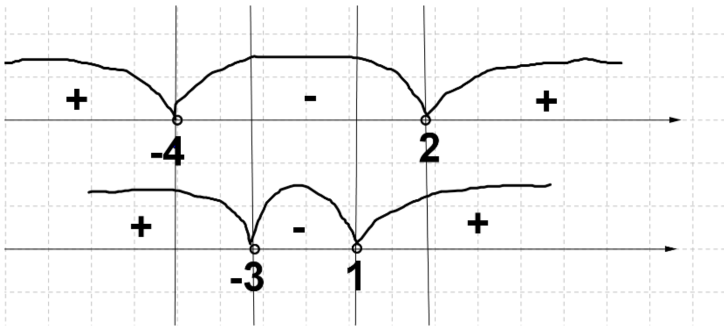
$\Leftrightarrow \begin{cases} x^2+2x-1=t \\ \frac{-t^2+9t-14}{(t+1)(t-1)}>0 \end{cases} \Leftrightarrow \begin{cases} x^2+2x-1=t \\ \frac{t^2-9t+14}{(t+1)(t-1)}<0 \end{cases} \Leftrightarrow \begin{cases} x^2+2x-1=t \\ \frac{(t-2)(t-7)}{(t+1)(t-1)}<0 \end{cases} \Leftrightarrow$

$\frac{(t-2)(t-7)}{(t+1)(t-1)}<0 \Leftrightarrow$



$\Leftrightarrow \left[\begin{array}{l} -1<t<1 \\ 2<t<7 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} -1<x^2+2x-1<1 \\ 2<x^2+2x-1<7 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} x^2+2x-2<0 \\ x^2+2x>0 \end{cases} \\ \begin{cases} x^2+2x-8<0 \\ x^2+2x-3>0 \end{cases} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \begin{cases} -1-\sqrt{3}<x<-1+\sqrt{3} \\ \left[\begin{array}{l} x>0 \\ x\le -2 \end{array}\right. \end{cases} \\ \begin{cases} (x+4)(x-2)<0 \\ (x+3)(x-1)>0 \end{cases} \end{array}\right. \Leftrightarrow$

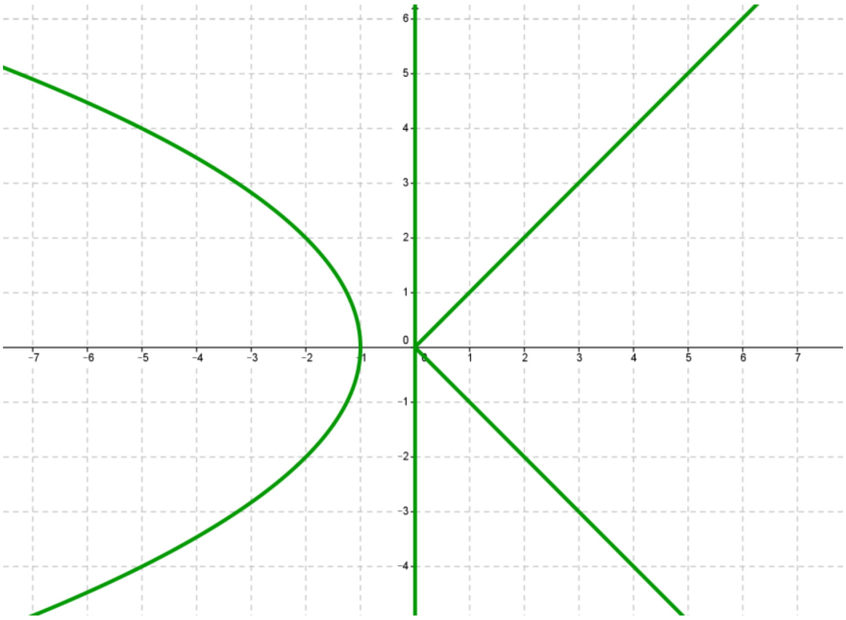




$\Leftrightarrow \left[\begin{array}{l} -4<x<-3 \\ -1-\sqrt{3}<x<-2 \\ 0<x<-1+\sqrt{3} \\ 1<x<2 \end{array}\right.$ Ответ: $\left(-4;-3\right)\cup\left(-1-\sqrt{3};-2\right)\cup\left(0;-1+\sqrt{3}\right)\cup\left(1;2\right)$.

Изобразите на координатной плоскости множество точек, координаты которых удовлетворяют заданному условию:

- $(y^2+4x+4)\big(\big|2x-|y|\big|-|y|\big)=0 \Leftrightarrow \left[\begin{array}{l} y^2+4x+4=0 \\ \big|2x-|y|\big|-|y|=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{y^2}{4}-1 \\ (2x-|y|)^2=y^2 \end{array}\right. \Leftrightarrow$

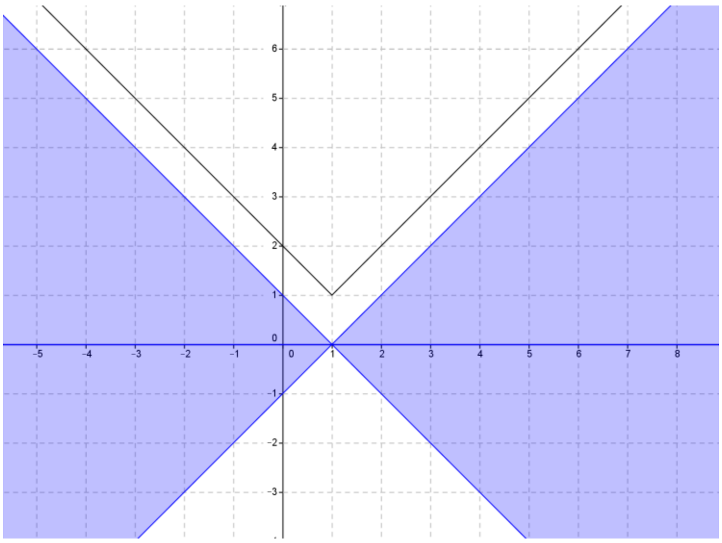


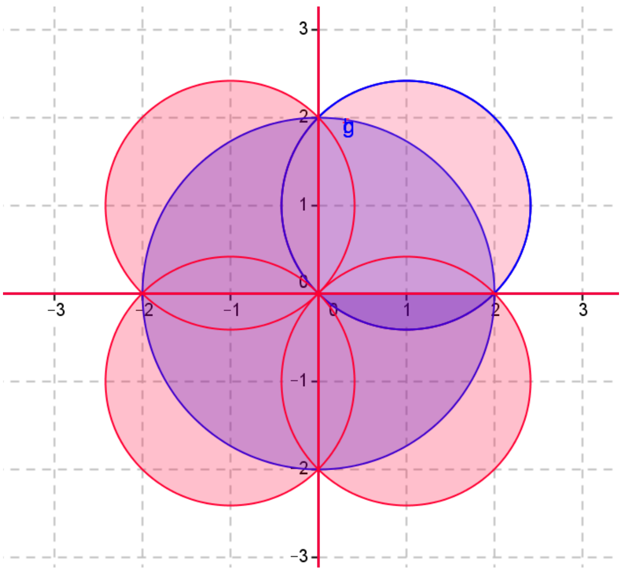
$$\Leftrightarrow \left[\begin{array}{l} x=-\frac{y^2}{4}-1 \\ (2x-|y|-y)(2x-|y|+y)=0 \end{array}\right. \Leftrightarrow$$

$$\left[\begin{array}{l} x=-\frac{y^2}{4}-1 \\ 2x-|y|-y=0 \\ 2x-|y|+y=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\frac{y^2}{4}-1 \\ \begin{cases} y\ge 0 \\ y=x \end{cases} \\ \begin{cases} y\le 0 \\ x=0 \end{cases} \\ \begin{cases} y\ge 0 \\ x=0 \end{cases} \\ \begin{cases} y\le 0 \\ y=-x \end{cases} \end{array}\right.$$

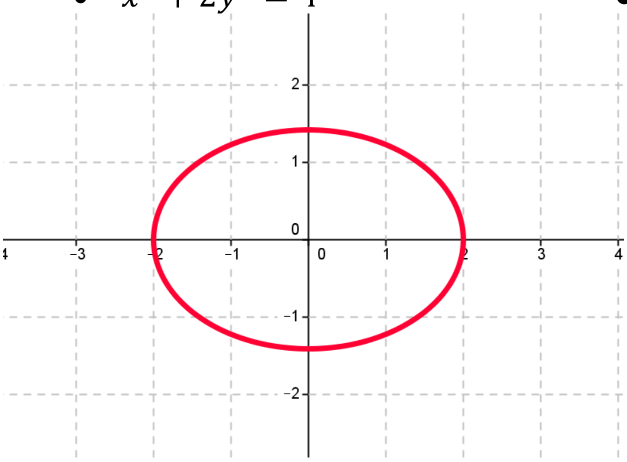
- $|y|\le\Big|\big||x-1|+1\big|-1\Big|$;

- $\begin{cases} x^2+y^2\le 4, \\ x^2+y^2\le 2|x|+2|y| \end{cases} \Leftrightarrow \begin{cases} x^2+y^2\le 4, \\ (|x|-1)^2+(|y|-1)^2\le 2 \end{cases}$

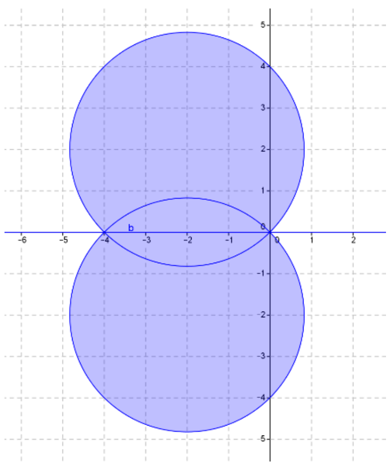




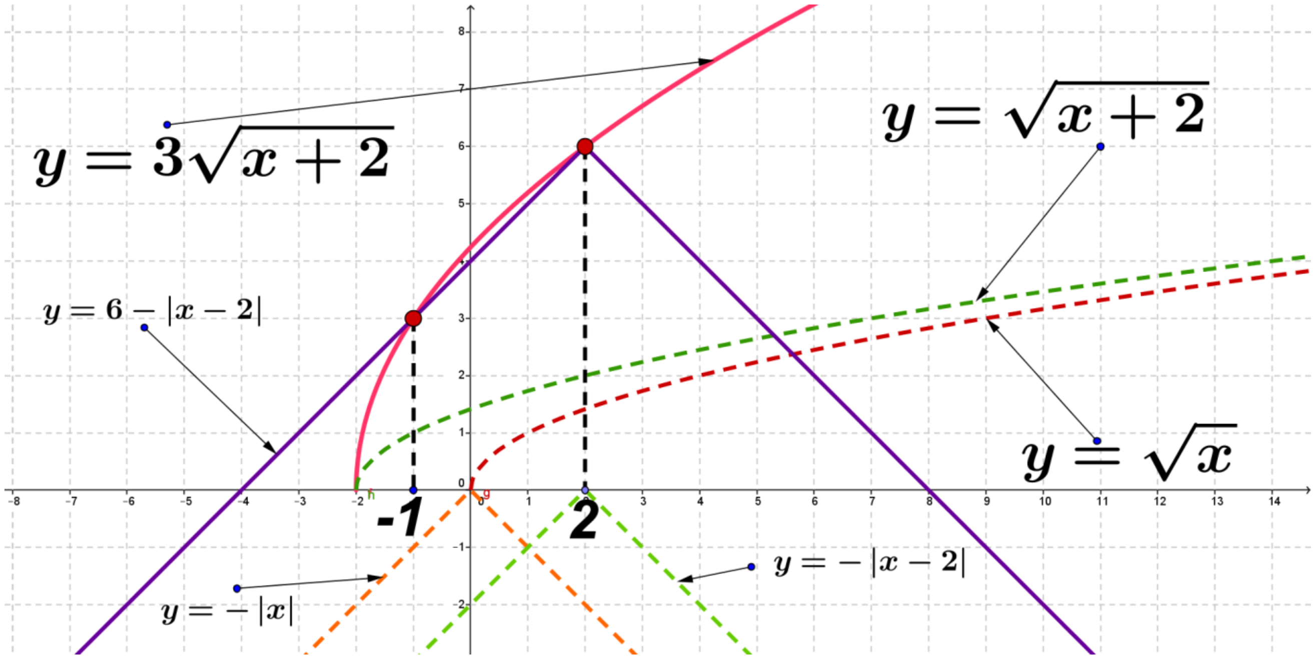
- $x^2+2y^2=4$



- $x^2+y^2+4(x-|y|)\le 0 \Leftrightarrow (x+2)^2+(|y|-2)^2\le 8$

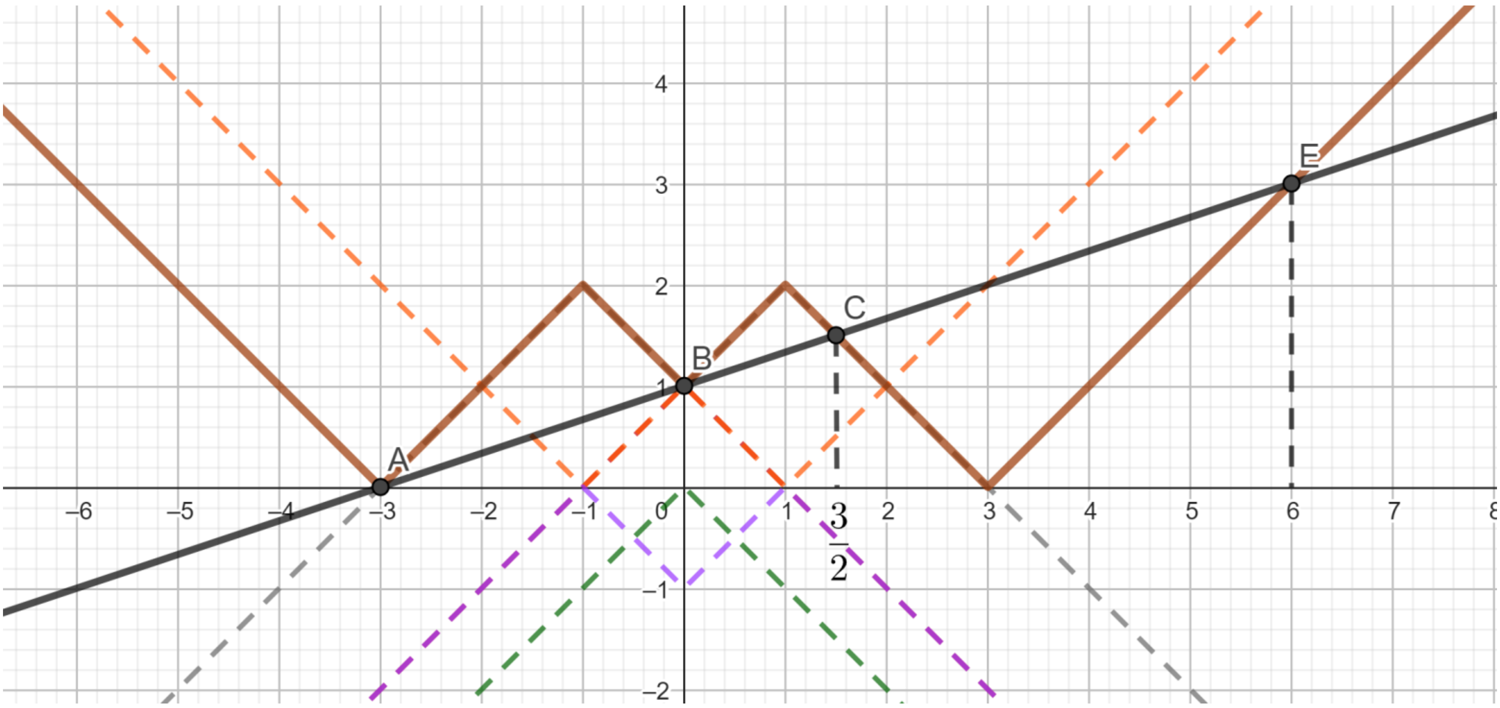


- **Найдите множество решений неравенства:** $3\sqrt{x+2} \le 6 - |x-2|$.



Ответ: $[-2;\ -1]; x = 2.$

- **Найдите множество решений неравенства:** $|2 - |1 - |x||| > \frac{1}{3}x + 1$.



Ответ: $(-\infty;\ -3) \cup (-3; 0) \cup \left(0; \frac{3}{2}\right) \cup (6; +\infty)$.